

Math Strategies

Teaching Math to Young Children

Grades PreK and K

RECOMMENDATION 1

Teach numbers and operations using a developmental progression.

RECOMMENDATION 2

Teach geometry, patterns, measurement, and data analysis using a developmental progression.

RECOMMENDATION 3

Use progress monitoring to ensure that math instruction builds on what each child knows.

RECOMMENDATION 4

Teach children to view and describe their world mathematically.

RECOMMENDATION 5

Dedicate time each day to teaching math, and integrate math instruction throughout the school day.

This document provides a summary of recommendations 1–5 from the WWC Practice Guide *Teaching Math to Young Children* (Frye et al., 2013).

Content Area	Level of Evidence	Grades
Math	 Moderate	PreK K

RECOMMENDATION 1

Teach numbers and operations using a developmental progression.

Understanding what skills and knowledge children already possess is the starting point for instruction. A developmental progression can provide a road map of the next steps. For example, when teaching numbers and operations, teachers should ensure that children proceed through each level of the developmental progression. At each level, children should begin by practicing with small sets of objects and progress to larger sets until they master the skills and knowledge at that level.

Example of a Piece of Developmental Progression for Number Knowledge

Subitizing (Small-Number Recognition)	Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of a quantity (e.g., two eyes, two hands, two socks, two shoes, two cars) labeled with the same number word, as well as nonexamples labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three. A child is ready for the next step when, for example, they can see one, two, or three stickers and immediately—without counting—state the correct number of stickers.
Meaningful Object Counting	Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is called the cardinality principle). A child is ready for the next step when, for example, if given five blocks and asked, "How many?" they count by pointing and assigning one number to each block: "One, two, three, four, five," and recognizes that the total is "five."

Counting-Based Comparisons of Collections Larger Than Three	Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., “seven” items is more than “six” items because you have to count further). A child is ready for the next step when they are shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., “nine” bears is more).
Number-After Knowledge	Familiarity with the counting sequence enables a child to have number-after knowledge—i.e., to enter the sequence at any point and specify the next number instead of always counting from one. A child is ready for the next step when they can answer questions such as, “What comes after five?” by stating “five, six” or simply “six” instead of, say, counting “one, two, . . . six.”
Mental Comparisons of Close or Neighboring Numbers	Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that “nine” is larger than “eight”). A child has this knowledge when they can answer questions such as, “Which is more, seven or eight?” and can make comparisons of other close numbers.
Number-After Equals One More	Once children can mentally compare numbers and see that “two” is one more than “one” and that “three” is one more than “two,” they can conclude that any number in the counting sequence is exactly one more than the previous numbers. A child is ready for the next step when they recognize, for example, that “eight” is one more than “seven.”

Note. Taken from Table 3 on page 13 of the practice guide (Frye et al., 2013).

Strategies

1. First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them.

Instructional strategies from the examples:

- Build children’s understanding of quantity through subitizing activities.
- Have children identify sets that contain an equal number of objects without counting.
- Build children’s ability to compare sets by having them find sets of objects that contain the same number of objects and those that do not (e.g., “three” and “not three”).

Children should be able to determine the number of objects in a small set without counting. This is known as *subitizing*. During classroom transitions, teachers may find small sets of objects (five or less) in the classroom and ask, “How many [object name] do you see?” After children can successfully identify three objects, they should be able to understand related sets that have the same number of objects. For example, three pencils and three erasers are two sets with the same number of objects. Once children have experience in recognizing sets of objects containing similar items, teachers can progress to sets with dissimilar items (for example, a set of three containing a pencil, a crayon, and an eraser). When developing subitizing, children may overgeneralize the term “three” or “four” to mean “many,” so teachers should identify a set of three objects as “three” and a set of four objects as “not three” to help children recognize the difference. Teachers can then challenge children to find sets of objects around the classroom that are “three” or “not three.” See Example 1 on page 16 of the practice guide referenced on the last page of this document.

2. Next, promote accurate one-to-one counting as a means of identifying the total number of items in a collection.

Instructional strategies from the examples:

- Build on children’s ability to subitize to help them develop one-to-one object counting, beginning with organized sets, then moving to unorganized sets.
- After successfully counting a set of objects, help children develop cardinality—recognizing that the last counting word represents the number of objects in the set.
- Capitalize on children’s counting errors to strengthen their counting ability.

One-to-one counting occurs when children count with number words in a consecutive order to determine the number of objects in a set, using only one number name for each unique item. For example, a child counting a set of pens points at a pen and says, “One,” then points at the next pen and says, “Two,” and finally points at the last pen and says, “Three.” Children should begin with small sets (one to four objects). Teachers should help them in realizing that the last number they count is the total number in a set. Then, children can progress to larger sets (four to 10 objects). Teachers should also demonstrate that order does not affect the result by using objects around the classroom and counting them in different ways. For example, when counting pens, no matter which pen children start with, they will reach the same result.

Example Counting Activity: The Hidden Stars Game

Objective

Practice using one-to-one counting and the final number counted to identify “how many” objects.

Materials:

- Star stickers in varying quantities from one to 10, glued to 5-by-8-inch cards
- Paper for covering cards

Directions: Teachers can tailor the Hidden Stars game for use with the whole class, a small group, or individual children. Show children a collection of stars on an index card. Have one child count the stars. Once the child has counted the stars correctly, cover the stars and ask, “How many stars am I hiding?”

Early Math Content Areas Covered

- Counting
- Cardinality (using the last number counted to identify the total in the set)

Monitoring Children’s Progress and Tailoring the Activity Appropriately

- Work with children in a small group, noting each child’s ability to count the stars with accuracy and say the amount using the cardinality principle (the last number counted represents the total).
- When children repeat the full count sequence, model the cardinality principle. For example, for four items, if a child repeats the count sequence, say, “One, two, three, four. So I need to remember four. There are four stars hiding.”
- Have a child hide the stars while telling him or her how many there are, emphasizing the last number as the significant number.

Using the Activity to Increase Math Talk in the Classroom

Ask, “How many?” (e.g., “How many blocks did you use to build your house? How many children completed the puzzle?”)

Note. Taken from Example 2 on page 18 of the practice guide (Frye et al., 2013).

Errors are to be expected when children learn how to count. Common counting errors include sequence errors, coordination errors, and keeping-track errors, as well as skimming and not recognizing cardinality. In sequence errors, children confuse the sequence of numbers. In this case, teachers might ask the children to sing the number sequence. If children are skipping specific numbers, teachers should focus on practicing that part of the sequence. Coordination errors involve children labeling an object with more than one word or pointing to an object without counting it.

Teachers might correct these errors by encouraging children to slow down and count each object once. When children make keeping-track errors, they may count an object twice. Teachers can help children differentiate counted objects from uncounted objects by having them pick up the counted objects and place them to the side. For more information on common counting errors and recommendations on how to remedy them, see Table 4 on page 19 of the practice guide.

3. Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities.

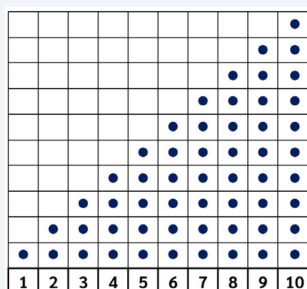
Instructional strategies from the examples:

- Initially present children with sets in which one is obviously larger, then move to sets that are more equal in size that require counting or subitizing.

Children can progress in making meaningful comparisons of sets by, for example, identifying “more” and “fewer.” First, teachers should present children with two sets of objects, one of which is obviously larger, and ask them which set has more or fewer objects. Next, teachers can demonstrate that the further in the counting sequence children count, the larger the numbers become. The sample chart below provides a visual of this increase in size or quantity and can be helpful. Real-world examples, such as counting to determine who has more points in a game, can also help children increase their ability to compare quantities.

Also helpful is knowing what number comes next without counting (number-after relations). Teachers can reinforce this knowledge by asking children such questions as “This is the sixth pen, so the next pen will be how many?”

Sample Cardinality Chart



Note. Taken from page 20 of the practice guide (Fry et al., 2013).

4. Encourage children to label collections with number words and numerals.

Instructional strategies from the examples:

- Label sets of objects with representations of quantity, numeral, and number words to help children connect the different representations.

Numerals are a way to represent quantity. To help children understand this, teachers should pair number words with numerals. For example, “3,” “three,” and “three objects” have the same meaning. Teachers can label sets of objects that children see in the classroom. For example, a set of three pens can be labeled “3, • • •, three.” The three dots provide scaffolding and represent the quantity for children who do not recognize numerals yet.

Example Activity: Numerals and Dots Game

Objective

Match numerals with corresponding quantities.

Materials needed:

- One set of 20 cards: 10 cards with numerals from 1 to 10 along with the corresponding number of dots, and 10 cards with pictures of objects (the numbers of objects corresponding to a numeral from 1 through 10).
- For even more advanced play, once children are proficient at numerals 1–10, teachers can create cards for numerals 11–20.

Directions: Half of the cards have a numeral and dots to represent the amount (e.g., the numeral 3 and three dots) on one side, and the other half have pictures of collections of objects on one side (e.g., three horses, four ducks). The other side of each card is blank. The cards are placed facedown, with the numeral cards in one area and the picture cards in another. A player chooses one numeral card and one picture card. If they match, then the player keeps those cards. Play continues until no further matching cards remain. The player with the most cards wins the game.

Early Math Content Areas Covered

- Numeral recognition.
- Corresponding quantity.
- If the objects in the pictures on the cards are in different orders, it can help reinforce the idea that appearance does not matter when it comes to numbers.

Monitoring Children's Progress and Tailoring the Activity Appropriately

- Play the game with a small group of children, noting each child's progress in practicing and achieving the objectives.
- This game can be played with children who are not familiar with numeracy concepts.
- Use fewer cards, lower numbers, or cards with dots to scaffold. As children gain proficiency with the concepts, increase the number of cards and the size of the numbers.

Using the Activity to Increase Math Talk in the Classroom

- Before asking, "How many?" ask, "How can we find out how many?"

Note. Taken from Example 3 on page 22 of the practice guide (Frye et al., 2013).

5. Once children develop these fundamental number skills, encourage them to solve basic problems.

Instructional strategies from the examples:

- Provide opportunities for children to explore the effects of adding or removing one object from a set using counting strategies.
- Once children have developed some facility, present situations where the final result is hidden.

Children should proceed to develop an understanding of the effects of altering the number of objects in a set as they prepare for math problem-solving. To start, children can remove or add one object, recount, and review how the number of objects has changed. They can also use counting strategies in problem-solving activities in the classroom, such as counting the number of groups to determine how many whiteboards to hand out.

After children have had opportunities to explore adding and subtracting objects from sets, teachers can move to situations where the final results are hidden from view. For example, teachers can show children a set of four pens, cover the pens with a cloth, take one pen from underneath the cloth, and then ask the children to determine how many pens are left under the cloth. Once the children decide how many objects are left, teachers can remove the cloth and have the children count to see if they solved the problem correctly. Snack time is also a great opportunity for children to apply counting skills. Teachers can ask, "How many will you have after you eat one snack item?" or, "How many will you have after your friend gives you one snack item?" to have children problem-solve without seeing the end set.

Potential Roadblocks



I want to provide strong math foundations for my children, but I am not really comfortable with math myself.

Suggested Approach. Teachers who feel less comfortable with math should base classroom projects on real-world examples. Setting up a toy store in the classroom provides a more comfortable setting in which to integrate lessons. Any activity that is of interest and involves counting presents an opportunity to build children's math skills.



Each child in the class is at a different level in the developmental progression I am using to guide instruction.

Suggested Approach. Teachers might split children into groups, using the developmental progression to create groups of children at a similar level. Dividing children into groups allows teachers to assign tasks based on each group's level of proficiency. Teachers can also create groups containing children at diverse levels of proficiency. This allows for children at a higher level to model a skill for others.



A child is stuck at a particular point in the developmental progression.

Suggested Approach. If children are stuck, they have likely not yet mastered a skill from an earlier level in the developmental progression. Teachers should use the developmental progression to help identify the unmastered skill and provide opportunities for children to practice it further before returning to the point at which they were stuck.

Content Area	Level of Evidence	Grades
Math	 Minimal	PreK K

RECOMMENDATION 2

Teach geometry, patterns, measurement, and data analysis using a developmental progression.

Understanding what skills and knowledge children already possess is the starting place for instruction. A developmental progression can provide a road map of next steps. To ensure that children have early opportunities to experience a wide range of math content, teachers should use a developmental progression to expose them to geometry and data. Teachers should ensure that children proceed through each level of the developmental progression. Helping children build understanding beyond numbers and operations increases their likelihood of success in later math.

Strategies

1. Help children recognize, name, and compare shapes, and then teach them to combine and separate shapes.

Instructional strategies from the examples:

- Have children find and name shapes in the world around them.
- Identify critical attributes of shapes, focusing on precise mathematical language.
- Provide both examples and non-examples of shapes.
- Present children with opportunities to combine and/or separate shapes to construct other shapes.

Teachers should begin by helping children find and name shapes in their own environments. Once children have developed confidence in naming shapes, they should be challenged to name the most important characteristics or “critical attributes” of each shape using standard vocabulary. For example, a critical attribute of a triangle is that it has three sides. Teachers should also point out characteristics that are not critical attributes. For instance, teachers should note that all sides of a triangle do not need to have equal length.

To solidify children’s understanding, teachers should provide both examples and non-examples of shapes. A non-example is a shape that lacks one or more of the critical characteristics. For a shape activity that highlights critical attributes, see Example 4 on page 29 in the practice guide referenced on the last page of this document.

Once children understand the fundamentals of shapes, teachers should ask them to explore how shapes can be combined and separated. For example, combining two identical squares can create a rectangle, or cutting a triangle across the middle may make two triangles.

Combining Two Identical Squares Will Make a Rectangle



Note. Taken from Figure 4 on page 28 of the practice guide (Frye et al., 2013).

2. Encourage children to look for and identify patterns, and then teach them to extend, correct, and create patterns.

Instructional strategies from the examples:

- Guide children to identify basic repeating patterns in activities and in the classroom.
- Introduce errors into patterns and challenge children to identify and correct the error.
- Build children's ability to extend patterns by having them predict what would come next.

First, teachers should ask children to experiment with basic repeating patterns, such as having them select a pattern by which everybody in the class will line up to go to lunch (for example, hot lunch, bag lunch, hot lunch, bag lunch, hot lunch, bag lunch). Once children understand simple patterns, they should be challenged to construct more complex ones (for example, hot lunch, hot lunch, bag lunch, hot lunch, hot lunch, bag lunch, hot lunch, hot lunch, bag lunch). Teachers can then challenge children to notice patterns in the classroom, such as tiles on the floor, stripes on clothing, bricks on a wall, or even seasons in the year. Teachers should also introduce errors to children to further challenge their ability to recognize and detect patterns.

After children demonstrate an understanding of patterns, teachers should ask them to use their understanding to predict what comes next. Teachers can give children a string of beads that follow a pattern and ask them to continue the pattern along the string.

Teachers can introduce complexity by first asking children to create a pattern based on instructions (two blue beads and then two yellow beads) and then proceeding to introduce new colors or characteristics (small or large beads). Lastly, children should design patterns themselves. See Example 5 on page 31 in the practice guide for a sample activity to create and extend patterns.

3. Promote children's understanding of measurement by teaching them to make direct comparisons and to use both informal or nonstandard (e.g., the child's hand or foot) and formal or standard (e.g., a ruler) units and tools.

Instructional strategies from the examples:

- Use simple examples to help children identify differences in measure between objects (e.g., direct comparison of length to determine which is longer).
- Provide opportunities for children to measure with both formal and informal units.

Teachers should begin with simple examples to highlight differences in measurement among objects and show how to directly compare them. For example, teachers can present children with two different-sized crayons, ask which one is longer, and then guide them to directly compare the lengths by placing the crayons side by side. Once children demonstrate an understanding of direct comparison, teachers can provide them with tasks such as ordering a set of four to five crayons from shortest to longest. Teachers should reinforce measurement vocabulary when making comparisons.

Examples of Vocabulary Words for Types of Measurement

Type of Measurement	Examples of Vocabulary Words
Length	Long, longer, longest; short, shorter, shortest
Size	Small, smaller, smallest; big, bigger, biggest
Temperature	Warm, warmer, warmest; cold, colder, coldest
Time	Early, earlier, earliest; late, later, latest
Weight	Heavy, heavier, heaviest; light, lighter, lightest

Note. Adapted from Table 5 on page 32 of the practice guide (Frye et al., 2013).

Next, teachers can challenge children to measure objects using informal or nonstandard units. For example, children can use their hands to measure the length of a piece of paper and express the measurement in numeric units (the paper is three hands long). After experience with informal units, children should be introduced to standard measurement units (inches, feet, ounces, pounds) and use standard measurement tools (rulers and scales) to measure objects. Starting with nonstandard measurement before moving to standard measurement reinforces that nonstandard measurement may give different results, while standard measurement does not. Teachers can ask multiple children to measure a length (such as the distance between two desks) using both standard and nonstandard units and compare the results. Standard measurement does not change between children, while nonstandard does. Everyday measurements, such as changes in temperature, time, weight, and height, provide opportunities for children to apply and expand their knowledge.

4. Help children collect and organize information, and then teach them to represent that information graphically.

Instructional strategies from the examples:

- Use counting and sorting physical objects as a way to help children begin to see visual representations.
- Encourage children to identify and discuss differences between two sorted sets.
- Guide children to use visual representations (e.g., tally marks) to represent the number of objects counted in a set.

Teachers should begin familiarizing children with the concept of grouping and visually representing information by asking children to count and sort tangible objects (for example, blocks, crayons) and abstract concepts (for example, 4-year-olds and 5-year-olds). Teachers should challenge children to identify characteristics that lead to grouping objects or individuals (for example, children with different pets, the shape of blocks). Children should recognize all characteristics that differentiate sets (shape, size, color) and then count the number of objects or individuals in each set. The two goals are to identify the characteristics that represent each set and compare the number of items in each set. Next, teachers should ask children to visually represent the information they collect, such as in graphs. Teachers should begin with simple tallies and progress to more complex graphs. For a sample activity for teachers, see Example 6 on page 34 in the practice guide.

Potential Roadblocks



It is challenging enough to cover everything I need to cover in a day without having to think about four more early math content areas.

Suggested Approach. Teachers can cover multiple math areas during one lesson. For example, they can ask children to find a collection of objects, count the items, and arrange them in a pattern, all in one lesson. Math games that can be played during transitions or downtime, such as “I spy,” can help teachers find time for math learning.



Some children are struggling with basic vocabulary skills or are being exposed to English for the first time.

Suggested Approach. Teachers can use visual representations of vocabulary concepts. Or, when multiple children in a classroom speak the same non-English language, teachers can assist English-speaking children in learning to count in their classmates’ native languages. Songs and fingerplays are also helpful for learning math vocabulary. Having children arrange materials or draw to display answers can also help to overcome the language gap.

Content Area	Level of Evidence	Grades
Math	 Minimal	PreK K

RECOMMENDATION 3

Use progress monitoring to ensure that math instruction builds on what each child knows.

Progress monitoring is a helpful method for making sure that math instruction is deliberate and useful for children. Teachers can monitor progress to tailor lessons and instruction based on children's current math skill levels. This method helps to ensure that children are receiving math instruction that is difficult enough so that they are always learning.

Strategies

1. Use introductory activities, observations, and assessments to determine each child's existing math knowledge, or the level of understanding or skill they have reached on a developmental progression.

Instructional strategies from the examples:

- Use introductory activities to determine children's ability in working with a new concept.
- Carefully observe children during a math activity, asking questions that require children to think out loud and describe their problem-solving process.
- Use formal assessments to guide planning and instruction.

To begin progress monitoring, teachers should find each child's level of skill and knowledge in math through introductory activities, observations, and formal assessments. Teachers can use introductory activities to present new concepts and determine what children can complete independently. For example, after instructing children on shapes, teachers can include an activity in which the children cut out shapes such as circles and triangles from magazines. Teachers can then ask the children to discuss the shapes and their different sizes. Such activity allows teachers to see whether the children know shapes and can talk about them.

Observations include addressing specific math competencies through activities and watching children in their process of completing the activities. While children are engaged in the activities, teachers can ask questions to see if they can verbalize their process for completing the activity. If children can complete the activity correctly and explain their process verbally, they can move on in the lesson.

Teachers can plan instruction and lessons by looking at children's overall performance on formal assessments to identify their skill levels. Teachers can also look at children's answers to specific questions or in specific sections of assessments for a deeper understanding of their knowledge and skills. This information can be valuable in selecting appropriate goals for instruction.

2. Tailor instruction to each child's needs, and relate new ideas to their existing knowledge.

Instructional strategies from the examples:

- Use information collected from Step 1 above to guide instruction.
- Build activities based on the next level of development and connect them to children's interests.

Teachers should use learnings from introductory activities, observations, and formal assessments to identify where children's knowledge and skills place them in the developmental progression. Doing so will help teachers determine the next steps in the learning process and create instructional activities aligned to the next levels. For example, when children demonstrate they can use subitizing to determine which set of objects has more (for example, a set of four has more than a set of three), they can use meaning counting to determine which collection contains more. Once children can count a set of 10 objects, teachers can include a set of 11 or more objects to increase the difficulty. See Table 3 on page 13 of the practice guide for a sample developmental progression related to number knowledge.

Teachers should not only create activities aligned to the next level in the learning progression but also connect new knowledge to children's interests. For example, if children like art, teachers can create activities that include drawing, such as having children draw eight dogs and 10 dogs and then describe which set has the most dogs.

Children might be at different levels in a learning progression, so it's helpful to group them by level for some activities. For example, some children might be able to count sets of 10 objects, whereas others might be able to count sets of eight. Teachers can group children who can count to 10 and group those who can count to eight. Then, they can observe the groups and increase the difficulty of the activity as the child's ability levels rise.

3. Assess, record, and monitor each child's progress so that instructional goals and methods can be adjusted as needed.

Instructional strategies from the examples:

- To understand children's learning, use repeated cycles of implementation, assessment, and planning.

Teachers can use progress monitoring to assess children's progress through introductory activities, observations, and formal assessments. Progress monitoring involves first selecting an activity using a developmental progression, then repeatedly cycling through a process of implementing the activity, assessing child's levels (using the developmental progression), and planning or selecting additional activities. Teachers might use a checklist such as the one below to monitor progress during a sample activity.

Progress Monitoring Checklist

Activity: Which set has the most triangles?

Set 1:



Set 2:



Child	Date	Counted Correctly?	Decided Correctly?	Errors Made
Suzy	November	Yes	No	Selected set 1 as having more triangles
Billy	November	No	Yes	Counted eight twice for set 2

Note. Adapted from Example 8 on page 40 of the practice guide (Frye et al., 2013).

When assessments show that children's math knowledge and skills are growing, teachers can plan activities, following the developmental progression, that are increasingly more difficult and continually assess children's math levels as they complete the new activities.

Potential Roadblocks



How can I maintain order in the classroom when breaking the class into small groups?

Suggested Approach. When children are in small groups, classroom behavior can sometimes become chaotic and noisy. There are three things to think about when forming small groups. First, group children strategically to avoid social conflicts. If children of mixed abilities are working together in groups, ensure there is the right mix of ability levels. Second, develop activities that build on children's interests. Using small groups enables teachers to present more challenging activities to some children so that they do not become bored. Finally, plan adult assistance to facilitate independent and adult-supported learning for all groups. One strategy for managing groups is to use round-robin learning centers. While one group is meeting with the teacher, other groups are productively engaged in different learning centers. There should be one center for each group not meeting with the teacher. The teacher then is free to focus on one small group at a time.



I am already required to give standardized assessments. Can I use my existing assessments to tailor instruction?

Suggested Approach. Teachers can review the assessments to find questions that apply to the particular skill they would like to target. Then, they can use those questions to gauge where children are and at which level to target activities. If teachers receive feedback on how children in their classroom are performing on a standardized assessment, they can fit this feedback in with the developmental progressions to determine which areas need more focus and when children can move on to higher-level skills. If there are assistant teachers, aides, or other adults available in the classroom, teachers can ask them to share in observing children and keeping brief checklists of children's ability levels.



What if I do not have required assessments, or the assessments do not fit well with the skills that are targeted in the developmental progression?

Suggested Approach. Teachers can use a developmental progression to develop an activity that will provide information about the child's skill level. For example, teachers can develop a checklist of numerals from 1 to 20 and use magnetic numerals or a numeral bingo game to assess the child's ability to recognize numerals. Teachers can generate checklists for counting collections, naming shapes, identifying patterns, sorting, and many other math skills.

Content Area	Level of Evidence	Grades
Math	 Minimal	PreK K

RECOMMENDATION 4

Teach children to view and describe their world mathematically.

Teachers can start by having children describe math they see and experience in the real world informally, using their own language. Once children are comfortable with informal representations and talking about math, they can move to more formal representations and vocabulary, with teacher support. Applying math to real-world situations while children play and work at school will help them develop a better understanding of the math concepts they are learning.

Strategies

1. Encourage children to use informal methods to represent math concepts, processes, and solutions.

Instructional strategies from the examples:

- Introduce new math concepts using terms and connections familiar to children.
- Focus on having children explain their mathematical thinking and reasoning in a way that makes sense to them, rather than initially focusing on correct mathematical language.

Teachers should begin math instruction by informally representing math concepts through connections to experiences, using recognizable terms and comparisons. For example, teachers should think about the vocabulary that children already know when teaching them addition. Children are likely familiar with the terms “take away” and “left.” Instead of using math terms for subtraction, teachers might say, “Suzy had four candy bars, and her mom took away one candy bar. How many candy bars does Suzy have left?”

Using Information Representations of Math Concepts

Concept	Informal Representation	Teaching the Concept
Whole number	“three”	Collections of blocks, dots, tally marks, fingers, or other countable objects can represent numerals. For example, when playing a game, use blocks to represent children’s scores so that everyone can track each player’s score.
Equal	“same number as” or “same as”	Provide opportunities for children to begin to recognize that collections that have the same number when counted are equal. For example, a collection of four plates is the same number as a collection of four cups.
Unequal	“more than” or “fewer than”	Point out that a collection is more (or fewer) than another if it requires a longer (or shorter) count. For example, seven is more than six because it requires counting beyond six.
Addition	“and” or “more”	Start with a collection and add more items to make it larger. For example, start with three crayons and add one more. Then ask, “How many?”
Subtraction	“take away” or “fewer”	Start with a collection and take away some items to make it smaller. For example, start with three crayons and take away one. Then ask, “How many?”

Note. Adapted from Table 6 on page 43 of the practice guide (Frye et al., 2013).

2. Help children link formal math vocabulary, symbols, and procedures to their informal knowledge or experiences.

Instructional strategies from the examples:

- Help children connect their informal language to formal mathematical language through modeling.
- Use formal mathematical terms throughout the school day.
- Connect children’s mathematical language to mathematical symbols.

Once children are comfortable with talking about and representing math concepts informally, teachers can begin introducing formal math concepts. Teachers can do so by connecting informal representations with formal math terms (for example, “more” with “addition”). To help children learn formal terms, teachers should be sure to use each term multiple times throughout the school day and also provide opportunities for children to repeatedly use the term. For example, when teachers read a book with the class, they can make connections to math terms by talking about the characters in the book, such as one character being younger or older than another.

Children must also learn to connect their informal math knowledge with formal math symbols (for example, “more” connects with $+$). An example activity might involve having children solve addition or subtraction problems with objects in the classroom. For more ways to connect informal knowledge to formal math symbols, see Table 7 on page 44 of the practice guide referenced on the last page of this document.

3. Use open-ended questions to prompt children to apply their math knowledge.

Instructional strategies from the examples:

- Use open-ended questions to help children build their ability to reason mathematically and use math vocabulary.

Teachers can use open-ended questions (“what,” “why,” “how,” and so on) to prompt children to think about math concepts and vocabulary. These questions allow children to apply what they know in answering a question. Open-ended questions should encourage children to use concepts and terms they are familiar with. For example, teachers might show a picture of two trees and ask, “What makes these two trees different?” This question invites multiple answers. One child might say, “The first tree is taller than the second tree.” Another child might say, “They are different kinds of trees.” Allowing for multiple responses lets children discuss concrete objects using math terms. Sample open-ended questions that teachers might use to encourage children to discuss math concepts are provided below. See the practice guide referenced on the last page of this document for more examples.

Examples of Open-Ended Questions

- What makes these the same/different?
- How can you tell how ____ (tall, short, long, wide) that is?
- How did you figure out how many ____ (blocks, buttons, shapes) there are?
- Why do you think this is ____ (taller, shorter, longer, wider) than that?

Note. Adapted from Table 8 on page 45 of the practice guide (Frye et al., 2013).

4. Encourage children to recognize and talk about math in everyday situations.

Instructional strategies from the examples:

- Provide opportunities for children to discuss connections between math concepts and the world around them.
- Have children explain their solution strategies out loud.
- Encourage children to think about other ways they might be able to solve a problem.

Teachers can strengthen children's math knowledge by providing opportunities for them to connect math concepts to the real world. For example, teachers might ask children for help in deciding the number of pencils needed for the classroom: "How should we figure out how many pencils we need for the classroom?" Teachers could then encourage the children to think about the solution path for the problem by discussing it out loud: "How did you think of that answer? What steps did you take?" Once the children have explained how they solved the problem, teachers can repeat the explanation out loud and then ask, "Are there different steps we could have taken to solve this problem?" If children are struggling to discuss their mathematical process, teachers can provide examples. This activity allows children to connect their math knowledge to problem-solving in everyday situations.

Potential Roadblocks



I'm not sure what types of open-ended questions are most effective for getting young children to think mathematically.

Suggested Approach. Use the strategies described in this recommendation to think about open-ended questions you might use. Start a math conversation by asking, “How can we figure this out? ” Once children have answered, prompt them to think about how they arrived at their answers by asking, “How did you think of that answer?” and “What steps did you take?” If there is more than one correct strategy, ask children, “Are there other steps we could have taken to solve the problem?”

Content Area	Level of Evidence	Grades
Math	 Minimal	PreK K

RECOMMENDATION 5

Dedicate time each day to teaching math, and integrate math instruction throughout the school day.

Each school day should have time set aside for math. Teachers should connect math concepts to familiar objects so children can see the relationship to everyday situations. It is also helpful to incorporate math concepts into lessons for other subjects so children begin to understand how those subjects are connected to math. Including math-related items, such as games, in the classroom can help children apply the math concepts they have learned in new contexts.

Strategies

1. Plan daily instruction targeting specific math concepts and skills.

Instructional strategies from the examples:

- Explicitly plan time during each day for children to develop math skills.
- Provide opportunities throughout the school day for children to apply their math concepts.

Children in the lower grades should have explicit time for developing math skills during each school day. During this time, children can learn specific math concepts through whole- and small-group activities. Then, teachers can provide opportunities throughout the remainder of the day for children to apply the math concepts they have learned. Children may be at different math levels, so teachers should consider how best to group them for each activity. For example, teachers might introduce a new math concept as a whole group and then divide children into smaller groups by math level to strengthen the learning through an activity or game.

Linking Large Groups to Small Groups

Objective	Understand the differences and similarities among triangles, rectangles, and squares.
Materials Needed	• Book: <i>Bear in a Square</i>, by Stella Blackstone • A variety of other objects (based on availability, but could include the following): ○ Large pieces of paper cut into various shapes for painting ○ Lunch trays and a small amount of sand ○ Geoboards with rubber bands
Directions: Large Group	Read the book in a large group, highlighting the names of all the shapes but focusing specifically on the difference between the number and length of sides and types of angles in triangles, rectangles, and squares.
Directions: Small Group	Once children are divided into small groups, highlight the number and length of sides and types of angles in each of the shapes the children create during activities like the one below. Children should be encouraged to use informal terms to describe the shapes. Provide paint, chalk, or other art materials so that children can add a stripe around the edge of a large paper cutout of a triangle or rectangle. Then, have the children continue to add more of the same shapes inside the original shape to create a design with concentric shapes.
Early Math Content Areas Covered	Geometry (shapes and attributes of shapes)
Integrating The Activity into Other Parts of the Day	Take a group walk outside to collect sticks of different sizes, and then use them to make and identify shapes.
Using The Activity to Increase Math Talk in the Classroom	When children locate a shape in the classroom environment, ask them to explain it to the group: “How can you tell that shape is a ___?” Prompt the children to identify the number and length of sides and type of angles.

Note. Adapted from Example 9 on page 49 of the practice guide (Frye et al., 2013).

2. Embed math in classroom routines and activities.

Instructional strategies from the examples:

- Look for ways to embed math into everyday classroom routines and activities.

Teachers can use opportunities within classroom routines and activities to allow children to practice the math concepts they have learned during math lessons. For example, teachers can have children count the number of children in attendance at snack time and then count out enough apples for each child to have one. Teachers could extend this activity and have the children keep track of the counts over two different days, then make comparisons between the two days. See Example 10 on page 50 of the practice guide for more detail about how math might be integrated into snack time.

3. Highlight math within topics across the curriculum.

Instructional strategies from the examples:

- Look for ways to integrate math into other content areas as it makes sense.

Teachers can provide children with opportunities to apply math concepts in other content-area lessons. Depending on where children are in their learning process, teachers can include time for counting, looking at shapes, or using other math concepts during instruction in other content areas.

Examples of How a Teacher can Incorporate Math into Other Content-Area Lessons

	Numbers and Operations	Geometry	Patterns	Measurements	Data Analysis
Literacy	<i>We All Went on Safari</i> , Krebs	<i>Bear in a Square</i> , Blackstone	<i>A Pair of Socks</i> , Murphy	<i>How Big Is a Foot?</i> Myller	<i>It's Probably Penny</i> , Leedy
Science	Count collections of natural objects.	Describe objects from nature in geometric terms.	Find and identify patterns in nature.	Measure the growth of a plant in the classroom each day.	Graph the amount the classroom plant grows each day.
Art	Count how many objects appear in a piece of artwork.	Identify shapes in artwork.	Use patterns to make pictures or frames for pictures.	Measure to make frames for art out of poster board or card stock.	Make a graph of the children's favorite colors.

	Numbers and Operations	Geometry	Patterns	Measurements	Data Analysis
Health and Safety	Count the length of time it takes to wash your hands.	Use traffic signs to recognize shapes.	Jump rope or play hopscotch with an alternating pattern.	Measure your body's growth over time.	Graph your height or foot size.
Social Studies	In a unit about families, order people by height or from youngest to oldest.	Identify squares, straight lines, curved lines, and other shapes on maps.	Study patterns in clothes from different parts of the world.	Make a map of the neighborhood, using measuring, geometry, spatial thinking, and positioning words.	Graph the size of the children's families.

Note. Adapted from Table 9 on pages 51–52 of the practice guide (Frye et al., 2013).

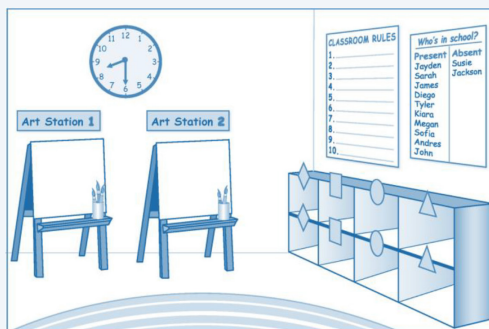
4. Create a math-rich environment where children can recognize and meaningfully apply math.

Instructional strategies from the examples:

- Provide accessible objects and tools throughout the classroom that are related to math concepts learned in the class.
- Encourage children to think about how to apply math concepts to everyday activities.
- Engage children in labeling tools and materials that can be used for math activities.

Teachers can create a classroom environment that is well-supplied for children to apply math. As an option, teachers can have accessible objects and tools in the classroom that are related to learned math concepts. Teachers might also have children apply math concepts to everyday activities and tools that are already part of the classroom routine, including number lists, blocks, beads, rulers, sorting bins, and so on. Children may not directly know how to use the tools to think about the math concepts they have learned, so teachers might model how to use them during instruction and group work. The tools should be labeled and placed in locations accessible to children. Teachers should also consider including children in the labeling process so they understand what each label means. Labeling is also an excellent opportunity for children to apply math concepts such as counting.

Example of a Math-Rich Environment in the Classroom



Note. Taken from Figure 7 on page 53 of the practice guide (Frye et al., 2013).

5. Use games to teach math concepts and skills, and to give children practice in applying them.

Instructional strategies from the examples:

- Use games to engage children in math.
- Align games with the math concepts children are learning in class and their level of understanding of these concepts.

Teachers can use games to immerse children in math concepts. Games might motivate children to engage in math in a fun way. Games should align with the math concepts being learned in class and with the children's levels. Curricula will often include games, but individual games can also be purchased elsewhere. Additionally, existing games can be used to apply math concepts. An example of a math game is provided below.

The Animal Spots Game

Objective	Practice One-to-One Correspondence and Cardinality
Materials needed	• Pictures of small animals or materials children can use to draw their own animals • Small circles of paper to use as spots • Glue • A die or spinner to determine the number of spots to place on each animal
Directions	Have each child draw the outline of an animal on a piece of paper, or provide handouts with large outlines of animals. Each child should take a turn throwing the die to determine how many spots to place on their animal. The children should count out the number of dots on the face of the die, and then they should choose the same number of “spots” from a bowl of paper circles in the center of the table. After children have selected the correct number of spots, they can glue them onto their animals. Teachers can tailor the Animal Spots game for use with the entire class, a small group, or individual children.
Early math content areas covered	• Counting using one-to-one correspondence • Cardinality
Monitoring children’s progress and tailoring the activity appropriately	• Observe the play, noting each child’s ability to count the number of dots on the die and count out the same number of spots from a larger pile. • Use one die or a spinner at the beginning; then, use two dice to increase difficulty.
Integrating the activity into other parts of the day	Have children count out objects from a larger set. For example, a child can choose 10 blocks for building or five shapes from a larger collection to use for a collage.

Note. Adapted from Example 11 on page 54 of the practice guide (Frye et al., 2013).

Potential Roadblocks



The school is on a limited budget and cannot afford to purchase many classroom materials or games.

Suggested Approach. To enhance instruction without having to spend money, teachers can use existing opportunities throughout the day, objects already in the school environment, and tools that others have created, such as those found online. When teachers or schools decide to purchase items, they should be strategic by thinking about what will best engage children in their current learning.



I am told that it is important to provide literacy-, science-, art-, and math-rich environments. It is difficult to keep all subjects in mind at all times.

Suggested Approach. When planning lessons, think about how to incorporate aspects of various content areas into each lesson. All content areas do not need to be incorporated at one time; math can be included in different subjects at different times. Be strategic about how to meet the learning goals of the day across content areas. See Table 9 on pages 51–52 of the practice guide for ideas.



I do not have my own space because multiple classes use the same classrooms throughout the day.

Suggested Approach. If teachers cannot easily change the classroom, they can consider how to use mobile objects to enhance the environment, such as transportable games or charts.



Families may wonder why their children are playing games in school.

Suggested Approach. Teachers can be strategic about choosing games to enhance math instruction and learning. They should connect games to specific learning objectives. Doing so will help families see the connections between the games and instruction and learning. Additionally, teachers should explain how important play is in the development of young learners.

Reference

Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children* (NCEE 2014-4005). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. <https://ies.ed.gov/ncee/wwc/PracticeGuide/18>